Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The ORIGIN *and* THEORY *of the* GOTHIC ARCH.

By the Reverend M. YOUNG, *D. D. F. T. C. D. and M. R. I. A.*

Read Nov. 7, 1789.

1. NOTWITHSTANDING the ſurpriſing things that have been effected by the architects of the middle ages, in raiſing ſuch ſtupendous piles on ſo ſlight an apparent ſupport as the pointed arch and ſlender Gothic pillar; it ſeems that their ſucceſſors have not, with all the care which the ſubject deſerves, conſidered how far any part of theſe great effects was to be attributed to the ſtructure of the arch. Whether it has been, that the ſuperior excellence of the Grecian architecture has afforded their genius ſo thorough employment, that they have not had leiſure to examine, with ſufficient accuracy, the works of our anceſtors of the Gothic age; or that their contempt of any thing that could originate amongſt Barbarians, as perhaps they concluded of this arch, has produced a neglect of that to which they might otherwiſe have attended with advantage; certain it is, that few, whether architects or mathematicians, have paid much regard to

this ſpecies of architecture. And thoſe who have incidentally made mention of it, have in general directed their enquiries to its origin rather than its principles.

2. The Saxon, Norman and Gothic ſtyles of architecture, though nearly related to each other, yet have their peculiar characteriſtics and diſtinguiſhing features. The Saxon and Norman architecture agree in this, that the form of the building is the ſame, the pillars round, ſquare or polygonal, and very ſtrong and maſſive, and the arches and heads of the doors and windows ſemicircular. Mr. Bentham thinks that the criterion of Norman architecture, by which it is diſtinguiſhed from the Saxon, is its ſuperior maſſiveneſs and enlarged dimenſions. The Saxon churches, he obſerves, were often elegant fabrics and well conſtructed, but generally of a moderate ſize, frequently completed in the ſpace of five or ſix years, or leſs time. The works of the Normans were large, ſumptuous and magnificent, of great length and breadth, and carried up to a proportionable height, with two and ſometimes three ranges of pillars, one over another, of different dimenſions, connected together by various arches, all ſemicircular, forming thereby a lower and upper portico, and over them a gallery. In the centre was a lofty tower, and ſometimes one or two added at the weſt end, as in the venerable cathedral at Hereford, which not long ſince has ſunk under the weight of years. But I am inclined to think there is a much more ſtriking difference between theſe two orders: I mean in the pillars. The Saxon columns, we know, were round, ſquare or polygonal, and very maſſive; but if we look into Doctor Ducarel's Norman antiquities, we ſhall find the Norman pillars, without exception, to be ſlender

and cluſtering. In England, therefore, the buildings erected in the time of the Norman princes were in a ſtyle compounded of both; to the ſimple and primitive Saxon adding the rich decorations and enlarged dimenſions of the Norman ſtructure. * The body or trunk of the pillars were uſually plain cylinders, or ſet off only with ſmall half columns united to them; but, to adorn them, they ſometimes uſed the ſpiral grove winding round them, and the net or lozenge work overſpreading them. The capitals were, in general, left plain without any manner of ſculpture, though inſtances occur, in ſome caſes, of foliage and animals on them. As to the arches, though for the moſt part plain and ſimple, yet ſome of their principal ones, as thoſe over the chief entrance at the weſt end, and others more expoſed to view, were abundantly charged with peculiar ſculpture, as the chevron work or zig-zag moulding; the embattled frette, the triangular frette, the nail-head, the billeted moulding, and the hatchet moulding. To adorn the inſide walls below, they had rows of little pillars and arches; and applied them alſo to decorate large vacant ſpaces in the walls without; they uſed alſo the corbel table; and the nebule, which was a projection terminating in an undulating line. To theſe marks of the Saxon and Norman ſtyle, we may add, that they had no tabernacles or niches with canopies, or pinnacles, or ſpires, or any ſtatues to adorn the outſide of their buildings, though ſometimes they had miniature relievo figures over the doors. † The marks which conſtitute the character of what is called the Gothic architecture, are its projecting buttreſſes, its pinnacles and ſpires, large ramified windows, niches, canopies

and

---

* Bentham's antiquities of Ely cathedral.

† Rious's architecture.

and ſculptured ſaints, the fretted roof, the cluſtering pillar, but above all, the pointed arch. And as plainneſs and ſolidity conſtitute the leading features in the Saxon and Norman buildings, ſo, on the other hand, the Gothic architecture is diſtinguiſhed by the lightneſs of its work, the boldneſs of its elevations, and the profuſion of its ornaments. So that the Saxon bears ſome ſimilitude as it were to the Tuſcan order; the Norman to the Doric and Ionic; and the Gothic to the Corinthian and Compoſite.

3. In our enquiries into the origin of the Gothic ſtyle, we meet with not leſs genius and fancy than has been diſcovered by the writers on the origin of the Grecian orders, but a much greater diverſity of ſentiment; there being not leſs than five different opinions held with regard to the riſe of this ſpecies of architecture: either that it was introduced from the eaſt by the Cruſaders, and ſhould therefore be called Saracenic; or borrowed from the Moors in Spain, and ſhould therefore be ſtyled Mooreſque; or derived from the ancient cuſtom of worſhipping in groves, where the eye being long accuſtomed to contemplate the arches formed by the branches of the trees that ſhaded their altars, it was natural, when covered buildings ſucceded to theſe groves of worſhip, that men ſhould endeavour to introduce ſome ſimilitude between them and thoſe places in which they had been accuſtomed ſo long to perform their religious ceremonies; and that accordingly we find not only the arches formed by the branches exactly imitated by the pointed arch, but the ſtems of the trees as accurately repreſented by the ſlender and cluſtering pillars. The elegance, ingenuity, and plauſibility of this opinion

have not failed to procure it the moſt general approbation. By others however it is maintained, that this arch originated from the interſection of circular arches in Saxon architecture; and others, laſtly, from the like interſection of circular arches in Grecian architecture.

4. WITH regard to the firſt opinion, which has been countenanced, or rather firſt advanced by Sir Chriſtopher Wren, ſome weighty objections obviouſly preſent themſelves. And firſt; inſtances may be produced of the exiſtence of the pointed arch in Europe antecedent to the Cruſades. * Dr. Stukely is of opinion, that the antient ſanctuary at Weſtminſter, in which the arches were pointed, was built by Edward the Confeſſor: "Nevertheleſs," ſays he, "I ſhall not be averſe to think it much older." † The church of Kirkdale has alſo the pointed arch, and is of the age of the Confeſſor. Croyland bridge in Lincolnſhire, which conſiſts of three Gothic arches, and is ſuppoſed to have been built in honour of the Trinity, is of an earlier date than the Cruſades, the time of its building being determined to be the year 860. Beſides, it does not appear, that what we call the Gothic is the prevailing ſtyle of architecture in the Eaſt; on the contrary, the ſtyle which predominates in that country is a corruption of the Grecian. This is a point ſo univerſally admitted, that Dr. Warburton maintains, that the Saxon architecture is an imitation of the Eaſtern.

---

* Archæol. vol. i.

† See Ledwich on Churches, Archæol. vol. viii.

Eaſtern. "When the Saxon kings," ſays he, "became Chriſtian, "their piety (which was the piety of the times) conſiſted in "building churches at home, and performing pilgrimages to the "Holy-land; and theſe ſpiritual exerciſes aſſiſted and ſupported "one another: For the moſt venerable, as the moſt elegant mo"dels of religious edifices were then in Paleſtine. From theſe "our Saxon builders took the whole of their ideas, as may be "ſeen by comparing the drawings which travellers have given "us of the churches yet ſtanding in that country with the "Saxon remains of what we find at home. Now the architec"ture of the Holy-land was entirely Grecian, but greatly fallen "from its antient elegance." See Pope's Eſſays, vol. 3, Ep. 4. And Mr. Ledwich, in his learned eſſay on the architecture of our Engliſh churches, advances very ingenious arguments to ſhew that the Saxon ſtyle had an Eaſtern origin, and produces a ſtriking inſtance from a Syriac MS. of the Evangeliſts, written A. D. 586, preſerved in the Medico-Laurentian library at Florence, which contains drawings of arches ornamented with every characteriſtic of the Saxon ſtyle. * Cornelius Le Brun has publiſhed many views of Eaſtern buildings, particularly of thoſe in the Holy-land; and in all theſe only one Gothic ruin, the church near Acre, and a few pointed arches occur, and theſe were built by the Chriſtians when in poſſeſſion of the country. In Sales Koran, the temple of Mecca is repreſented as built with ſemicircular not pointed arches. Neither did Pocock, Norden, or Shaw diſcover any traces of this ſtyle, as far as we can argue from

---

* Groſe's Antiquities, Pref.

from the drafts which they have given us in their travels. It appears therefore, that there is little or no reaſon for ſuppoſing our Gothic buildings imitations of the Eaſtern; for although the pointed arch may in ſome few places occur, yet it by no means is the characteriſtic of the Eaſtern ſtyle.

5. THE ſecond opinion, that the Gothic architecture was derived from the Moors, who introduced it with their victorious arms into Spain, from whence it was imported into the other countries of Europe, appears to be leſs tenable than the firſt; the form and decorations of the Mooriſh arch being very different from the Gothic, as may be ſeen in the drawings which are given us of Mooriſh buildings, particularly of the celebrated palace of Alhambra. See Swinburne's travels in Spain. See alſo in Houel's Pictureſque Voyage through the iſlands of Sicily, Malta and Lipari, a drawing of the prince of Palagonia's palace at La Bagaria, which is a Mooriſh building. Neither are there the leaſt traces of Gothic architecture in the drawings of the Mooriſh palaces in Les Delices d'Eſpagne *. In all theſe, and alſo in the Mooriſh caſtle of Gibraltar, the arches are either ſemicircles, or ſegments greater than ſemicircles, approaching to the horſe-ſhoe form, which is the genuine Mooriſh arch.

6. THE third opinion is that of Dr. Stukely, "The original "of all arts," ſays he, "is deduced from nature; and aſſuredly "the idea of this Arabian arch, (ſo he calls the Gothic) and ſlender

---

* Groſe's Antiquities, Pref.

“ ſlender pillar, is taken from the groves ſacred to religion, of “ which the great patriarch Abraham was the inventor.” This hypotheſis was adopted by Dr. Warburton, who has ſet it forth to full advantage in his notes on Pope's Eſſays. See Ep. 4. Againſt which however there lie ſome objections that do not admit of very obvious anſwers.

1ſt, When worſhipping in buildings of wood and ſtone ſucceeded to the cuſtom of worſhipping in groves, if the faſhion of theſe buildings had been ſuggeſted by the images preſented in groves, as this opinion ſuppoſes, then the moſt ancient ſtyle of religious buildings would have been Gothic. But this is contradicted by fact; for in the Eaſt, where worſhipping in groves firſt prevailed, the architecture is not of this primitive ſpecies, but is a corruption of the Grecian, in which the circular form of the arch is retained. And it is a known truth, that the moſt ancient religious buildings, of which we have any account, were not in that ſtyle which is called Gothic. But if ſo, this hypotheſis muſt be abandoned; for it is unreaſonable to ſuppoſe, that the arches in the firſt religious buildings which immediately ſucceeded the period of worſhiping in groves, ſhould not bear any reſemblance to thoſe which are repreſented by the interlacing branches of trees, when the minds of the architects were ſo ſtrongly impreſſed with the images of thoſe “ Leafy Cathedrals,” and yet, that the form of the arches, which were introduced ages after that cuſtom, and all its concomitant ſcenery had ſunk into utter oblivion, ſhould be derived from them.

2dly, Though it ſhould be granted, that the origin of Gothic architecture was derived from the cuſtom of worſhiping in groves, yet there does not appear to be any reaſon why this ſtyle ſhould therefore be introduced into military architecture. Yet we find, in fact, that, in England at leaſt, it has been introduced into military perhaps as early as into religious buildings. Edgar's tower at Worceſter, which is in the Gothic ſtyle, was the portal into the Saxon citadel, and was erected in the reign of Ethelred, the ſecond ſon of Edgar, anno 1005, as appeared from an ancient inſcription on a ſtone in the building which was taken down a few years ago: the inſcription may be ſeen in Mr. Green's hiſtory of that city. Doctor Littleton, dean of Exeter, in a diſſertation read before the ſociety of Antiquarians in 1757, has indeed ventured to affirm, that this tower is not older than the time of King John, on the authority of Mr. Habington; who ſays, "that in "*all likelyhood*, the figure of the King with croſſed legs repreſents "King John, who, anno 1215, took on him the ſign of the croſs "for the holy voyage; or King Richard the 1ſt, whoſe lyon's "heart ſo conquered the infidels." But he takes no notice, as is obſerved in Groſe, of the female figures on each ſide of the King, nor of the date. And as both theſe kings had each but one wife, the dean believes the figure really repreſents King Edgar, and the two female figures Ethelfleda and Ethelfrida. But if this be admitted, it ſeems to follow, that it muſt be of the antiquity which the date implied; for it is highly improbable, that in the reign of one of the Norman princes, a tower ſhould be erected to the honour of a Saxon king, who had died two hundred and fifty years before. But whether this building be altogether of as ancient a date as is aſſerted, or not, it is evident that the introduction of this arch into military buildings could not have ariſen from any deſire of imitating the original ſcenes of religious worſhip; it muſt therefore have taken its riſe either from a ſuppoſition of its ſuperior ſtrength, beauty, or other peculiar advantages: And this being once admitted, there is immediately aſſigned a more probable and ſufficient reaſon for its introduction into religious architecture alſo.

3dly, ONE of the principal arguments in favour of Doctor Stukely's opinion is, that the Gothic ſtyle comprehends not only the pointed arch but the ſlender and cluſtering pillar alſo; the one repreſenting the ſtems of the trees, the other the arches formed by the interſection of their boughs. "Could the arches be other-"wiſe than pointed," ſays Doctor Warburton, "when the work-"man was to imitate that curve which branches make by their "interſection with one another? Or could the columns be other-"wiſe than ſplit into diſtinct ſhafts, when they were to repreſent "the ſtems of a groupe of trees?" But this argument overturns the hypotheſis in confirmation of which it is advanced; for on this ſuppoſition the ſlender and cluſtering pillar ſhould be of the ſame date with the pointed arch; and they ſhould have been inſeparable companions in art as they are in nature, of which it is pretended, that this ſtyle is a cloſe copy. Now we know, that the pointed arch when firſt introduced into England, was not ſupported on ſlender pillars, but on the maſſive columns of the Saxon order, which were in faſhion at that time; and afterwards on the ſlender fluted pillars of the Normans, which were at length ſplit into diſtinct ſhafts, at a more advanced period of the Gothic ſtyle. So that, in England at leaſt, the pointed arch and cluſtering pillar have had different originals and periods of introduction. This obſervation ſeems to me effectually to overturn the hypotheſis we are at preſent examining, one main part of the fancied ſimilitude between Gothic architecture, and the ancient ſcenes of religious worſhip, being thus proved to have ſprung from a different ſource.

4thly, If the pointed arch had been intended to reſemble arcades formed by the branches of trees, we muſt ſuppoſe, that the ornaments of that arch would have had a cloſe affinity to their original. But on the contrary we find, that leaves or foliage conſtitute but an occaſional and accidental, not an eſſential part of the Gothic ornaments.

7. The fourth opinion on this ſubject is, that the Gothic arch has probably taken its riſe from thoſe arcades we ſee in the early Saxon and Norman buildings, where the wide ſemicircular arches connecting the alternate pilaſters, croſs and interſect each other, and form at their interſection a narrow and ſharp pointed arch. But there does not appear ſufficient grounds for our attributing the origin of this arch in general to the interſection of Saxon circles; for if ſo, the Gothic architecture would have had a Saxon original: but we find it in countries where no other traces of the Saxon ſtyle occur. Thus the pointed arch diſcovers itſelf amongſt the Mooriſh arches in the famous palace of Alhambra; and not only the pointed arch but fluted pillar alſo is to be found

in the church of St. Mamas, at Morfou, in the iſland of Cyprus. See Drummond's travels. Georgio Vaſari, in his proem to the lives of the Painters, ſays, that in the time of Theodoric, (that is, about the year 500) the architecture in Italy was of that manner called Tedeſco or Teutonic *. And the baptiſtery at Piſa, built in the year 1063, was ſubſequent to the period of Gothic architecture in that country, in which, though built in the Grecian ſtyle, we yet diſcover evident traces of the Gothic, which was now giving way to a better faſhion: But, at that period, we are certain, that the architecture in England was not Gothic. And as we find the uſe of the pointed arch in countries ſo widely diſtant, we muſt look for ſome more general principle of its introduction than the interſection of Saxon arches.

8. The

---

* After ſpeaking of the five Grecian orders, he thus deſcribes the ſtyle which he calls Tedeſco: "There is another ſpecies of architecture called Tedeſco, which in its ornaments and proportions is very different from the antique and from the modern; nor is it uſed at preſent by good architects, but is avoided as barbarous and monſtrous; being entirely deficient in order, ſo that it may be rather called diſorder and confuſion. The doors are ornamented with ſlender pillars, and twiſted like vines; which cannot have ſtrength to ſuſtain a weight, however light it may be. And thus by all their mouldings and other ornaments, they made a jumble of tabernacles one upon another, with ſo many pyramids, and points, and leaves, that they are not only unable to ſuſtain a weight, but ſcarcely even to ſtand; and they have more the appearance of being made of cards, than of ſtones and marble. And in theſe works they made ſo many ſudden tranſitions, breaks, little brackets, and turns, that they marred the proportion of their works: and often went to ſuch a height, that the top of the door touched the roof. This manner was invented by the Goths, who, after they had deſtroyed the ancient fabrics, and the architects were ſlain in war, introduced the arches of the fourth point, and filled all Italy with this monſtrous ſtyle of building." *Vaſari's Lives of the Painters and Architects, vol. i. p. 65.*

8. THE fifth opinion is that which has been given by Mr. Barry, in his very ingenious and elegant *Enquiry into the real and imaginary obſtrúctions to the arts in England*, where he endeavours to ſhew us the ſucceſſive corruptions of architecture, as they grew out of one another, by which it was led inſenſibly from the Grecian to the Gothic taſte. " There are at the dome of Viterbo," ſays he, " and at St. Mark's, Venice, ranges of columns, from " which regular arches, of half a circle, interſect one another, " by being made to ſpring from every third column, which gives " a Gothic arch between every two." And he farther adds, that before the great nich, which has been cut through the attic of the pantheon, there are two columns which are remarkable, as the flutes are more than a diameter of a circle deep. This outrayed a little, by ſinking the flutes deeper, and leſſening their number, he thinks, led to the Gothic bundle of columns.

THAT the Saxon, Gothic, Mooriſh and Eaſtern ſtyles of building are but various corruptions of the Grecian, ſeems evident: The ſimilar diviſion of the column into pedeſtal, ſhaft, and capital, the imperfect acanthus, and the faint traces of the Ionic volute which we ſometimes meet with, will not ſuffer us to doubt of this; but that the Gothic arch derived its origin from the identical circumſtance in Grecian architecture here pointed out, ſeems by no means ſufficiently evident. For St. Mark's, Venice, according to the teſtimony of Vaſari, was built towards the concluſion, not commencement of the period of corrupt architecture. So that if the pointed arch was indeed ſuggeſted in this manner, which however is extremely uncertain, it muſt rather have been derived from the church of St. Sophia at Conſtantinople, which was built in the 20th of Conſtantine, that is, in the year 344, and was the original from which the plan of St. Mark's, Venice, was taken. Although the Gothic architecture commenced ſo early amongſt the Goths and Longobards, and that the true taſte began to revive about the year 973, yet attachment to inveterate cuſtoms and the beauty of the ſtyle itſelf prevented it from falling into entire diſuſe for ſome centuries after; according to the obſervation of Piraneſi, in his work *Della magnificenza d'architettura de' Romani*, "la "Greca (architettura) poi non pote eſſer portata in Egitto, ſe "non ſotto il regno de' Tolommei: quantunque per altro, ſic- "come difficilmente ci diſtacchiamo dai coſtumi inveterati, é "coſa piu probabile, che a que' tempi ſi confondeſſe l'una coll' "altra, di quel che l'Egizia foſſe affatto poſta in diſuſo. In quella "guiſa appunto che noi vediamo la chieſa di S. Agoſtino, tanto "tempo dopo eſſere ſtati ſcacciati d'Italia i Goti e i Longobardi, "cioé l'anno 1483 di Criſto, eſſere ſtata fatta alla Greca, e inſieme di quell forma ſtraniera, la quale era ſtata in uſo in Italia "per tanto tempo." We are further to obſerve, that when it is ſaid, that the deepening of the flutes in the Grecian columns led to the Gothic cluſtering pillar, it is not neceſſary to ſuppoſe, that the Gothic ſtyle, in this reſpect, was directly derived from the pure Grecian, without paſſing through any intermediate gradations. In England it is more than probable that the fluted column, and its offspring the cluſtering pillar, was of Norman deſcent, as may be conjectured from the drawings of Doctor Ducarel; which Norman ſtyle, we muſt allow, was but a corruption of the Grecian, thus at length propagated into England.

And that we, of theſe countries, are not in any wiſe indebted to the Norman builders for the pointed arch itſelf, cannot I think be very poſitively aſſerted, when we conſider that the windows of the palace of William the conqueror at Caen were all pointed. So that although the pointed arch might have exiſted in England prior to this period, yet it was not till after the Norman conqueſt that it became the prevailing faſhion. This circumſtance of the turn of the arches in the palace at Caen Dr. Ducarel ſeems to have forgot when he tells us, * page 102, that the pointed arches were not introduced till near the end of the twelfth century. Italy has long been the ſchool for the fine arts, and as the Gothic architecture, from the teſtimony of Vaſari, ſeems firſt to have originated there, it is probable, that architects bringing home with them from thence the idea of the pointed arch, it aſſumed in different countries the garb of that into which it had been admitted. Thus the Gothic buildings in Germany, England, Spain, though they agree in the form of the arches, differ widely in their decorations.

" THERE iſſued from the hands of the maſters of thoſe times," ſays Vaſari, " thoſe fantaſtical and rude things which appear, even " at this day, in what remains of their works. The ſame thing " happened in architecture, for it being neceſſary to build, and " the form and good manner being entirely loſt by the death of " the artiſts, and the deſtruction of their works, they who gave " themſelves up to ſuch ſtudies, did not build any thing which for " order

---

* See his Norman Antiquities.

" order or meaſure had grace, either in the deſign or in any pro-
" portions. Whence new architects aroſe, who from their barba-
" rous notions produced that manner of building which by us is
" now called *Tedeſco*, who did ſome things which afforded rather
" a ſubject of laughter to the moderns, than praiſe to them;
" until afterwards better architects found out a better form, and
" ſomewhat like unto the good antique. In this ſtyle are the
" oldeſt churches in Italy, but not antique, which were built by
" them, as by Theodoric, King of Italy, a palace in Ravenna, one
" in Pavia, and another in Modena, but in the barbarous man-
" ner, and rather rich and large, than well deſigned or of good
" architecture. The like may be ſaid of St. Stephen's in Rimini,
" of St. Martin's in Ravenna, and of the temple of St. John the
" Evangeliſt, built in the ſame city about the year of our ſalva-
" tion 438; of St. Vitale, which was built in the year 547, and,
" in fine, of many other monaſteries and temples built after the
" Longobards. All which buildings, as has been ſaid, are great and
" magnificent, but of moſt incorrect architecture; and amongſt
" theſe are many abbies in France, built to St. Benedetto; and the
" church and monaſtery of Mount Caſino; the temple of John
" the Baptiſt at Monza, built by that Theodolinda, queen of
" the Goths, to whom Pope Gregory wrote his dialogues; in
" which place ſhe cauſed to be painted the ſtory of the Longo-
" bards. Afterwards, in Florence, architecture recovering a little,
" the church of the Holy Apoſtle, which was built by Charlemagne,
" though ſmall, was in a very beautiful manner. The architecture
" of this church was ſuch, that Pippo di Ser Brunnelleſco did not
" diſdain to make uſe of it as a model for the church of the

" Holy Ghoſt, and that of St. Lorenzo in the ſame city. The " ſame may be ſeen in St. Mark's, Venice, which was begun " under the Doge Juſtinian, and finiſhed in the Grecian manner " in the year 973. And thus things went on till the year 1140. " In the ſame Greek manner, and at the ſame time, were the " ſeven abbies, which the Count Hugo, Marquis of Brandenburgh, " cauſed to be made in Tuſcany; and other buildings, " which ſhew, that architecture kept its ground a little, but " much corrupted, and very different from the good ancient " manner; of which many old palaces bear teſtimony, built in " Florence, after the ruin of Feſole, of Tuſcan workmanſhip, " but in the barbarous ſtyle, in the meaſure of the doors, and " very long windows, and in the curviture of the ſharp arches " of the fourth point, in the turn of the arches, according to " the practice of the ſtrange architects of thoſe times. Afterwards " in the year 1013 the art ſeems to have recovered its " vigor a little; in the rebuilding of the beautiful church of " St. Miniato, in which it appears, that the Tuſcan artiſts endeavoured " to imitate in the doors, windows, columns, arches, " cornices, as much as they could, the good ancient order, &c." I have been thus full in this extract from Vaſari, as he is ſo expreſs as to the period and manner in which this Gothic ſtyle of building firſt originated, and is ſo competent a judge of ſuch a ſubject.

Mr. Ledwich indeed, in the eſſay above referred to, is of opinion, that the Gothic arch had been known and uſed many centuries before the Gothic power was eſtabliſhed. But the two inſtances which he produces, as he himſelf obſerves, are not perfectly Gothic, but thoſe arches called *contraſted*, which have never been aſcribed to the Goths and Longobards: and we are further to obſerve, that they are not ſuppoſed to repreſent an actual building, but the ornaments of a cibarium only.

But in truth it ſeems extremely nugatory and fruitleſs to endeavour to aſcertain the ſpecific accident which gave birth to the pointed arch; many almoſt innumerable different circumſtances could be aſſigned from which it might have originated; and who would venture confidently to adopt one in excluſion of the reſt? One may, perhaps, appear more probable than another; but diſcoveries do not always ariſe from thoſe circumſtances, which after they have been made, might have been ſuppoſed moſt naturally to have led to them. Who, for inſtance, after ſtudying the works of Newton, could have conjectured, that his great theory of the univerſal gravitation of matter had been ſuggeſted by the falling of an apple from a tree?—The mind indeed may ſometimes amuſe itſelf with excurſions into the aerial regions of fancy and conjecture; but to find reſt, it muſt return to the firm ground of fact. However entertaining therefore our ſpeculations may be on the poſſible cauſes which gave riſe to the idea of the Gothic arch, we can be ſaid to reaſon only when we direct our enquiries to the actual properties of the arch, which might have recommended its introduction into architecture, after the faſhion of ſuch an arch had been conceived. In other caſes, as in the circle, ellipſe, and catenaria, the nature of the curve alone is ſuppoſed ſufficient for that purpoſe; why then in the Gothic arch are all theſe conſiderations to be paſſed over? Certainly if it be poſſeſſed of any peculiar advantages, it ſeems unreaſonable to ſuppoſe, that they were entirelyun known or diſregarded; and that it is really poſſeſſed of ſuch, or at leaſt has been ſuppoſed to be ſo, which is ſufficient in the preſent caſe, we may conclude from what has been ſaid by Monſ. Gautier on this ſubject, in his *Traitè des Ponts et des Chemins;* "The Goths," ſays he, "who ſucceeded the good taſte of the Roman architecture, "erected bridges in many parts of France with pointed arches, "deſigning by that means to diminiſh the lateral preſſure, both "in public and private buildings, and alſo in the churches of "that period, which ſtill remain." And Sir Chriſtopher Wren obſerves, "that the ſharp-headed arch riſes with little centering, "requires lighter key-ſtones, and leſs butment, and yet will "bear another row of double arches riſing from the key-ſtone." The idea of the pointed arch might indeed have been firſt ſuggeſted in the manner deſcribed by Mr. Barry, or perhaps by the perſpective of groin arches; but the mere conception of ſuch a form, would not, I ſhould imagine, have been a ſufficient motive with architects to introduce it into their moſt magnificent buildings, and where it was to ſuſtain the greateſt weights. I ſhall therefore now proceed to enquire into the theory of this arch, and endeavour to aſcertain its relative ſtrength as compared with circular and elliptic arches, when in a ſtate of perfect equilibration; and alſo to determine the aberration from a true balance, which is generated by the horizontal termination of the ſolid building erected on it; from whence we ſhall be enabled to form ſome conjecture, whether the theory of the arch itſelf

may

may not be juſtly enumerated amongſt the cauſes, to which we owe its introduction into architecture.

9. THE equilibration of arches, which has been ſo well treated of by Mr. Emerſon in his *Eſſay on the Conſtruction of Arches*, and Mr. Hutton in his *Eſſay on Bridges*, may be aſcertained in a very ſimple manner, by conſidering the vouſſoirs, or arch-ſtones, as ſo many wedges urged by the incumbent weight, and endeavouring to ſplit the arch. Now the efficacy of a wedge depends on the magnitude of its vertical angle, the impelling force, and the reſiſtance to be overcome. On the firſt account the force of the vouſſoir, conſidered as a wedge, at any point of the arch, is directly as the radius of curvature of the arch at that point; on the ſecond account, it is as the ſquare of the ſine of the angle formed by the tangent to the curve, at the given point, with a vertical line; and on the third account, it is directly as the ſine of the ſame angle.

10. THE materials of which arches are compoſed are ſuppoſed to be of the ſame magnitude; and the joints of the ſtones muſt always be perpendicular to the curve, for every force acting againſt a ſurface acts in lines perpendicular to it; and therefore the force of the incumbent weight to rend the curve is to be reſolved into two, one of which muſt be perpendicular to the curve, and the other, of conſequence, in the direction of the tangent: now if this latter force were not perpendicular to the joints, that is, if the joints were not perpendicular to the curve, there would ariſe

a lateral preſſure, whoſe direction would not be along the tangent, and wanting a force to ſuſtain it, would deſtroy the equilibrium. Let E A B F, *r m n t*, repreſent two concentrical ſemicircles: alſo let A B, *m n*, be two vouſſoirs ſimilarly ſituated, whoſe ſides, being perpendicular to the curves, converge in the centre C: the forces of theſe vouſſoirs, conſidered as wedges, are inverſely as the ſines of their vertical angles, that is, becauſe of the equality of the arches, directly as the radii of curvature: And the like holds in any other curves whatſoever. Fig. 1.

11. Again, let H *h* be the invariable breadth of the vouſſoir, and G *g* H *h* the incumbent weight; this weight, becauſe the altitude G *h* is ſuppoſed given, is directly as the breadth *h k* of the incumbent column; that is, if *h* H be conſidered as radius, directly as the ſine of the angle *h* H *k*, formed by the tangent at the point H with the vertical line *g* H: but the force of this entire weight muſt be reſolved into two, one *g* K in the direction of the tangent, the other H K perpendicular to the curve, which is the force impelling the vouſſoir to ſplit the arch; and the line H K, which repreſents this latter force, becauſe *g* H is given, is the ſine of the angle H *g* K, which is equal to that formed by the tangent at H with the vertical line. Hence therefore, conjoining both theſe ratios, the force impelling the vouſſoir is as the ſquare of the ſine of the angle formed by the tangent to the curve, at the given point, with the vertical line.

12. Again, the wedge, impelled in a direction perpendicular to the curve, endeavours to ſplit the arch, and therefore move

one ſegment about the fulcrum *r*, and the other about the fulcrum *t*; and therefore the force of the vouſſoir, acting on the levers H *t*, H *r*, is directly as the perpendiculars *t p*, *r q*, let fall from the fulcrums on the line of direction H *q*, that is, directly as the ſine of the angle *t* C *p*, or H *g* K.

13. Hence therefore if a weight, like a wall, be incumbent on the arch E A B F, ſtanding in a vertical plane, if the height of the wall on any point be in a ratio compounded of the reciprocal triplicate ratio of the ſine of the angle formed by the curve and vertical line, and the reciprocal ſimple ratio of the radius of curvature in that point, all the vouſſoirs will endeavour to ſplit the arch with equal forces, and therefore will be in perfect equilibrium with each other. On this general propoſition depends the theory of the conſtruction of arches.

14. But ſince the materials with which arches are conſtructed are not of infinite ſtrength, there muſt be a certain degree of preſſure, which will rend even an arch whoſe parts are in perfect equilibrium. Now the ſtrength of an arch, in its different points, is as the greateſt weight which it is able to bear on thoſe points without breaking; that is, in a ratio compounded of the reciprocal triplicate ratio of the ſine of the angle formed by the curve and vertical line in its different points, and the reciprocal ſimple ratio of the radius of curvature in the ſame points. On this principle the comparative ſtrength as well of different arches as of the ſame arch in its different points may be eaſily aſcertained.

15. Let B D, conſidered as variable, repreſent the ſpan of the Gothic arch B E D, ſpringing from the points B, D; and the invariable line C D the radius of the circle by which the Gothic arch is deſcribed. On B D deſcribe the ſemicircle B H D; the ſtrength of the ſemicircular arch B H D at H is to the ſtrength of the ſemicircular arch A G D at G, as C D to K D, by Art. 10. But, ſince the relative ſtrength of arches is to be determined by comparing them in their weakeſt points, (for if an arch fails in any one point, the whole falls to ruin) the ſtrength of the ſemicircular arch A G D will be to the ſtrength of the Gothic arch, as the ſtrength of the ſemicircle at G to its ſtrength at E, that is, as the greateſt weights theſe points are able to bear when in equilibrium; that is, reciprocally as the cube of the angle I E F contained by the vertical line I E and the tangent to the circle at E, or its equal E C K, that is, as the cube of E K to the cube of radius, Art. 14. Therefore, *ex æquo*, &c. the ſtrength of the ſemicircular arch B H D at H, is to the ſtrength of the Gothic arch of the ſame ſpan, deſcribed with the radius C D, as $GC \times EK^3$ to $KD \times GC^3$, or as $EK \times AK$ to CD^2. Fig. 2.

16. Since the rectangle $EK \times AK$ vaniſhes at both extremities when K arrives either at A or D, there muſt be ſome intermediate ſtate where it is a *maximum*; to find this ſtate, let $CD = a$, and $CK = x$; $EK = \overline{a^2 - x^2}|^{\frac{1}{2}}$, and $AK = a + x$,

therefore the rectangle E K × A K $= \overline{a^2 - x^2}|^{\frac{1}{2}} \times \overline{a+x}$; whoſe fluxion $\overline{a^2 - x^2}|^{\frac{1}{2}} \dot{x} - \frac{x^2 \dot{x} - ax\dot{x}}{\overline{a^2 - x^2}|^{\frac{1}{2}}} = o$. Whence $x^2 + \frac{1}{2} ax = \frac{1}{2} a^2$, and $x = \frac{1}{2} a$. That is, the rectangle E K × A K is a maximum, when C K $= \frac{1}{2}$ C D: or, becauſe C D is conſtant, the ratio of the ſtrength of a ſemicircular arch to a Gothic arch of the ſame ſpan is greateſt when the ſubtenſe of half the Gothic arch is equal to its ſpan. In this caſe the ſtrength of the ſemicircular arch is to the ſtrength of the Gothic as 1299 to 1000, or 13 to 10, nearly.

If the radius of the Gothic arch be three-fourths of the ſpan, in which caſe it is called the ſharp arch of the fourth point, the ſtrength of a ſemicircle will be to the ſtrength of a Gothic arch of the ſame ſpan as 1257 to 1000. And if the radius be two-thirds of the ſpan, in which caſe it is called the ſharp arch of the third point, the ſtrength of the ſemicircle will be to the ſtrength of the Gothic arch of equal ſpan as 1210 to 1000.

17. Because the rectangle E K × A K exceeds the ſquare of the ſemidiameter when the point K biſects C D, and vaniſhes when K arrives at D, there muſt be ſome intermediate poſition, in which the rectangle E K × A K is equal to the ſquare of radius. To find the magnitude of K D in this caſe, let C K $= x$, and C D $= a$, as before; then E K $= \sqrt{a^2 - x^2}$, and A K $= a + x$; therefore $\overline{a+x} \times \sqrt{a^2 - x^2} = a^2$, and $x^4 + 2ax^3 - 2a^3x = o$; and if $a = 1$, $x^3 + 2x^2 = 2$. Of this equation two roots are impoſſible: The

third, or C K = ,839286755214, &c. That is, if the radius of the circle, by which a Gothic arch is deſcribed, be to the interval of the capitals from which it ſprings, as 1 to ,321426489572, the Gothic and ſemicircular arch of equal ſpan will be alſo of equal ſtrength.

18. HENCE therefore, a Gothic arch whoſe radius of curvature is equal to the interval between the pillars, which is the uſual manner of deſcription, is the weakeſt of all Gothic arches of that ſpan. According as the centre from which it is deſcribed moves on either ſide, in an horizontal line, the ſtrength of the arch increaſes: if it moves towards the middle point of the interval of the pillars, the ſtrength increaſes till the centre arrives at K, when the Gothic arch becomes a ſemicircle. But if the centre moves in the contrary direction, the ſtrength of the arch increaſes without limit.

19. MONS. GAUTIER therefore is too general in his aſſertion when he ſays, that the Gothic arch is ſtronger than the ſemicircle; as this is true only in certain caſes, to wit, when the radius by which it is deſcribed exceeds the ſpan in the ratio of 1 to ,321426489572, very nearly; in all other caſes it is weaker. We may obſerve alſo, that Sir Henry Wotton is not ſufficiently warranted in his cenſure of this arch, when he ſays, that "theſe "kind of arches, both for the natural imbecility of the acute angles, "as likewiſe for their uncomelineſs, ought to be exiled from all "judicious eyes, and left to their firſt inventors, the Goths and "Lombards, amongſt other reliques of the barbarous age." As to

its beauty or deformity, it being a matter not ſubject to mathematical argument, his criticiſm on that head cannot be brought to the like deciſive teſt as the former, and therefore we ſhall not pretend to controvert it.

20. THE depreſſed or ſcheme Gothic arch is deſcribed from centres, lying below the right line joining the capitals of the Fig. 3. pillars from which it ſprings: thus the arch B F D is deſcribed from the centres *m*, *n*, which lie below the horizontal line B D. Its ſtrength is eaſily computed by continuing the arch F D to the horizontal line R S which paſſes through the centres *m*, *n*: and then, proceeding as before, the ſtrength of the Gothic arch B F D will be to the ſtrength of a ſemicircular arch of equal ſpan, as $FG^3 \times KD$ to mS^4, that is, becauſe both F G and K D muſt always be leſs than *m* S, in a ratio of leſs inequality. And the ſtrength of the two Gothic arches B E D, B F D will be to each other in the direct duplicate ratio of their radii of curvature, and the inverſe triplicate ratio of the perpendicular heights of the arches above the lines of their centres. The ſtrength alſo of the Gothic arch of four centres may be eſtimated in the ſame manner.

21. BUT as the buildings erected on Gothic and ſemicircular arches are never ſo formed as to render them curves of perfect equilibration, but are generally terminated by an horizontal right line, it will be neceſſary to compare them in this reſpect; and we ſhall find, that though the ſtrength of the Gothic arch, when in

equilibrio, is in certain caſes greater than that of a ſemicircular arch of equal ſpan, yet a given altitude of ſolid building erected on the Gothic will in ſome caſes, but not in all, produce a greater aberration from a perfect equilibrium than in the ſemicircular. Let B N E D, B H D, be the Gothic and ſemicircular arches, whoſe Fig. 4. common ſpan is B D; let G R repreſent indifferently the extradoſſo of either, from which let fall the perpendicular G M on the ſpan, meeting the curves in N, *n*; and let *a* be the altitude of the ſolid building erected on each. If the Gothic arch were in perfect equilibrium, the height above N would be to *a*, as EK^3 to NM^3; and the height above *n* in the circle would be to the ſame *a* as HK^3 to nM^3 (Art. 13.). Now if an ellipſe were deſcribed with B D as a minor axis, and with E K as a ſemitranſverſe, it would fall without the Gothic arch, and meet the perpendicular G M ſomewhere between N and G, for the radius of the circle oſculating the ellipſe in B is equal to $\frac{EK^2}{BK}$ (Ham. Con. Prop. 17. L. 5.) that is, equal to the greater abſciſſa of the circle B N E, and therefore greater than the radius of the Gothic arch; and the oſculating circle falls without the ellipſe (Ham. Con. Prop. 15. L. 5.); but ſince the oſculating circle touches the ellipſe ſo intimately, that no circle can paſs between it and the ellipſe, the Gothic arch muſt fall within it; therefore the ratio of E K to N M is greater than the ratio of E K to *m* M, that is, than the ratio of H K to *n* M (Prop. 30. L. 1. Ham. Con.); therefore the height of the ſolid wall over N, in the Gothic arch, muſt be greater than over *n*, in the circle; and conſequently, when the line bounding the wall is not the extradoſſo of equilibrium but an horizontal line, there will be a

greater

greater deviation from a perfect balance in the Gothic arch. But the higher the Gothic arch the leſs will be the aberration; and it may, at length, be ſo far diminiſhed as to become leſs than the error in the ſemicircle, for the ordinates E K, N M, vary in the ſubduplicate ratio of their greater abſciſſæ; and theſe abſciſſæ, their difference M K being given, approach towards equality, as the radius of the circle or height of the arch increaſes; but the ratio of H K to *n* M is the limit of their variable ratio; ſo that though the height of the wall to be erected over the correſponding points in the Gothic and ſemicircular arch, in order to render them curves of equilibration, continually approach to equality, and can be brought nearer to an actual equality, by elevating the point of the Gothic arch, than by any aſſignable difference, yet the height of the building to be erected on the points of the Gothic arch will always be greater than in the ſemicircle. Now as E *p*, the difference between the ordinates N M, E K, is always greater than E *r*, the difference between the ordinates *m* M, E K, and theſe are always in the conſtant ratio of *n* M to H K, it follows, that by producing the ordinates *m* M, E K, their difference may be made greater than any aſſignable quantity; and at the ſame time the difference between the heights of the building to be erected on the points N, *n*, of the Gothic and ſemicircular arch, may be made leſs than any aſſignable quantity. It is evident therefore, that the defect of equilibrium over the point N, in the Gothic arch, may be made leſs than the defect over the correſponding point *n* of the ſemicircle, ariſing from the horizontal termination of the building erected on them. Thus let the difference of the heights of the wall over n and N, when the arches are in equilibrium, be equal to d; if Hv be to $Hv+d$, as HK to KE, Hv will be equal to $Er-d$. Then a being the altitude of the building over the crown of both arches, and Ng the height over the point n in the ſemicircle, when truly balanced, the defect of equilibrium in the Gothic arch will be $Ng+d-a-Ep$, and in the ſemicircle it will be $Ng-a-Hv=Ng+d-a-Er$; but Ep is greater than Er, and therefore the defect of equilibrium in the Gothic arch is leſs than in the ſemicircle. If Er be too ſmall, produce EK till $Er-Hv=d$, which is greater than δ, the difference of heights when EK is ſo produced: then, *a fortiori*, the deviation from a perfect balance will be leſs in the Gothic arch, as before. In all caſes the aberration will be leſs when the altitude over the crown of the arch is leſs; becauſe, when quantities are in a given ratio, the leſs the quantities themſelves are, the leſs will be their difference. And therefore, when the height of the building above the ſpring of the arch is given, the higher the arch the more it approaches to a perfect balance.

HENCE we may obſerve, that this arch was peculiarly adapted to the ſtyle of thoſe religious buildings, which were in faſhion in the middle ages, where the roof was to be raiſed to an extraordinary height, and no great weight immediately incumbent on the point of the arch, or where one tier of arches was to be raiſed over another. But when a very high building is to be erected upon a Gothic arch, the quantity of matter over the crown or vertex muſt be very much lightened by windows or other perforations, as was the practice with the Gothic architects.

22. If an ellipſe be deſcribed on the ſpan of a ſemicircular arch, its ſtrength, when in equilibrio, will be to that of a ſemicircular arch[*] reciprocally as the radii of curvature at the keyſtones

---

[*] Hence the ſtrength of an elliptic arch whoſe leſſer axis is perpendicular to the horizon, and of a portion of a circle of the ſame ſpan and altitude may be eaſily compared. For the radius of the circle paſſing through the points B E D is Fig. 5. equal to $\frac{EK^2+KD^2}{2\,EK}$, and the radius of the circle oſculating the ellipſe in D is equal to $\frac{KD^2}{EK}$; therefore the ſtrength of the ſegment of the circle is to the ſtrength of the ſemi-elliptic arch as $2\,KD^2$ to EK^2+KD^2, that is, always in a ratio of greater inequality. If to this we add, that the aberration from a perfect equilibrium in the ſemi-ellipſe, ariſing from the horizontal termination of the building erected on it, is very great, but of no great conſequence in any ſegment of a circle which contains leſs than 120°, (ſee Hutton's Eſſay on Bridges,) perhaps it may ſeem reaſonable to conclude, that portions of circles are in all caſes preferable to ſemi-elliptic arches, or thoſe curves of many centres, which of late have become ſo faſhionable in the conſtruction of bridges. In the celebrated bridge over the Sein, at Neuilly near Paris, conſiſting of five arches, each 120 feet opening, and riſing but 30 feet, the radius of curvature at the crown of the arch is 150 feet, and therefore is to the radius of a circular arch of the ſame ſpan, and riſing only to the ſame height, as 2 to 1; and conſequently the relative ſtrength of theſe arches, without taking into account the enormous error from a true equilibrium in the former, is in the reciprocal ratio of theſe numbers. And the ſtrength of a ſemi-ellipſe of the ſame ſpan and altitude is to the ſtrength of the curve uſed in the bridge of Neuilly as 5 to 4.

The radius of curvature of a circle oſculating the vulgar cycloid at its vertex is equal to twice the diameter of the generating circle; and the radius of a ſegment of a circle of the ſame opening and height is equal to the ſum of the ſquares of half the circumference and diameter applied to twice the diameter; therefore a cycloid is weaker than a circular arch of the ſame ſpan and height in the ratio of 11 to 16 nearly. To this we may add, that the accurate conſtruction of cycloidal arches, in ſtone, is almoſt impracticable, and therefore a conſiderable deduction ſhould be made from their ſtrength, on account of their imperfect execution. For theſe reaſons arches of this kind are not introduced into practice; though Drummond, in his travels, tells us, that thoſe of the bridge of the Holy Trinity, over the Arno, at Florence, are cycloidal. And indeed what is related of that bridge is conſiſtent with theory, for on account of its extreme weakneſs, it is found neceſſary to prevent waggons and other heavy carriages from paſſing over it.

ſtones (by Art. 14.); that is, as the ſemidiameter of the ellipſe which is perpendicular to the horizon to the ſemidiameter of the circle, (Prop. 17. L. 5. Ham. Con.) or as the height of the ellipſe to the height of the ſemicircle; that is, if B *m* E D be an elliptic arch, its ſtrength will be to that of the ſemicircle B H D, as Fig. 4. E K to K D; but the ſtrength of the ſemicircle is to the ſtrength of the Gothic arch B E D as E K × A K to CD^2; therefore, Fig. 2. *ex æquo*, *&c.* the ſtrength of the elliptic arch is to the ſtrength of the Gothic arch of equal height and ſpan as $EK^2 \times AK$, or $AK^2 \times KD$ to $CD^2 \times KD$, or as AK^2 to CD^2, or in the duplicate ratio of the ſum of radius and the coſine of the Gothic arch to radius, that is, in a ratio of greater inequality. And ſince K D is conſtant, A K increaſes in a higher proportion than C D; therefore in the elliptic arch, the ſtrength increaſes in a higher proportion, as the altitude increaſes, than in the Gothic. Thus in an elliptic and Gothic arch in which the heights and ſpans are equal, and the ſubtenſes of their halves equal to the ſpan, the ſtrength of the elliptic will be to that of the Gothic as 22½ to 10. And when the ſtrength of the Gothic arch is equal to that of a ſemicircle of equal ſpan, the ſtrength of an elliptic arch of equal height and ſpan exceeds it nearly in the ratio of 5 to 2 by Art. 17.

23. THE extradoſſo G R of an elliptic arch B E D may be thus in- Fig. 6. veſtigated: through any point N of the ellipſe draw the tangent N V meeting the axis K E produced in V; from K the centre let fall the perpendicular K A on the tangent; and draw K C the ſemiconjugate of

K N which paſſes through the point of contact; and N T the ordinate to the axis: Alſo let B K $= c$, K E $= t$, E T $= x$, K A $= p$, and K C $= g$; then K V $= \frac{t^2}{t-x}$, (Pr. 48. L. 1. Ham. Con.); and $p = \frac{tc}{g}$ (Cor. 1. Pr. 31. L. 1. Ham. Con.) therefore $\frac{t^2}{t-x} : \frac{tc}{g} :: 1$ (rad.) : Sine of the angle K V N or M N A, which is therefore equal to $\frac{c \times \overline{t-x}}{tg}$, and its cube equal to $\frac{c^3 \times \overline{t-x}^3}{t^3 g^3}$; alſo R, the radius of curvature at the point N is equal to $\frac{g^2}{p}$ (Cor. Pr. 18. L. 5. Ham. Con.) $= \frac{g^3}{tc}$; therefore R × Sin.^{3} of M N A $= \frac{c^2 \times \overline{t-x}^3}{t^4}$; and the height of the wall above the point N, which is reciprocally as R × ſin.^{3} M N A, will be directly as $\frac{t^4}{c^2 \times \overline{t-x}^3}$, which at the vertex E is $\frac{t}{c^2}$; and conſequently the height of the wall above N is to the height above E as $\frac{t^4}{c^2 \times \overline{t-x}^3}$ to $\frac{t}{c^2}$, or as t^3 to $\overline{t-x}^3$, that is, as E K^3 to N M^3; or as H K^3 to nM^3; therefore the heights of the wall above each point N of the ellipſe and the correſponding point n of the ſemicircle are equal. Conſequently when the altitude over the vertex of the arch is given, and the termination of the wall horizontal, there will be a leſs deviation from a true balance in the higher arch, that is, in the ellipſe; and the height over the point N is to a, the given height over E, as E K^3 to N M^3, therefore equal to $a \times \frac{EK^3}{NM^3}$; and the height over n equal to $a \times \frac{EK^3}{mM^3}$; and the difference equal to $a \times EK^3 \times \frac{mM^3 - NM^3}{NM^3 \times mM^3}$ which is greater than $a \times EK \times \frac{mM - NM}{NM \times mM}$, that is, greater than $a \times EK \times \frac{NM}{NM^2}$, which is greater than N M, the difference of the ordinates, for the altitude can ſcarcely be conceived to be ſo ſmall as the difference of the ordinates; conſequently there will be a leſs deviation from a true balance in the elliptic arch than in the Gothic.

24. It follows therefore, that an elliptic arch, whoſe tranſverſe axis is perpendicular to the horizon, is to be preferred to the Gothic arch for ſtrength; both becauſe its ſtrength exceeds that of a Gothic arch of equal ſpan and altitude, when both are in a ſtate of equilibration; and alſo becauſe a given altitude of building raiſed upon it and terminated by an horizontal line produces a leſs aberration from a perfect balance of the parts. It follows alſo that the Gothic arch, when carried up to a ſufficient height, is to be preferred to a ſemicircular arch for the ſame reaſons.

It ſeems unneceſſary to compare the ſtrength of the piers requiſite to ſuſtain elliptic and Gothic arches with the ſtrength of the piers of ſemicircular arches, as, from the great elevation of their vertexes, they are entirely unfit to be applied in bridges, where alone piers are introduced.

Face Page 88. Sci.

Fig. 1.

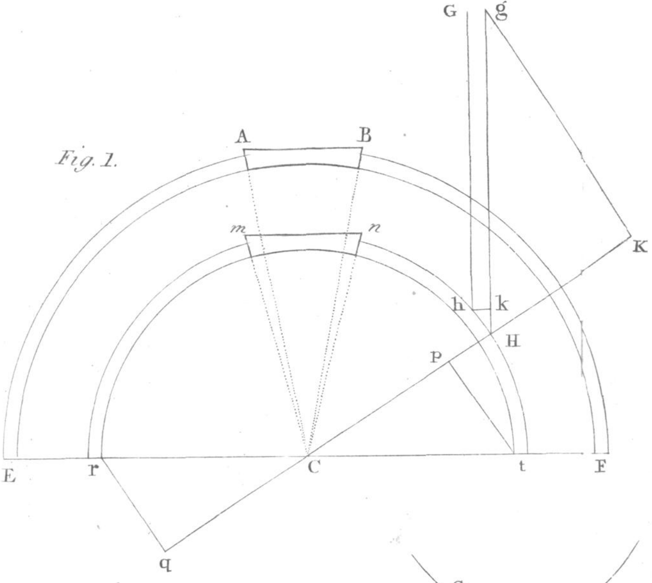


Fig. 2.

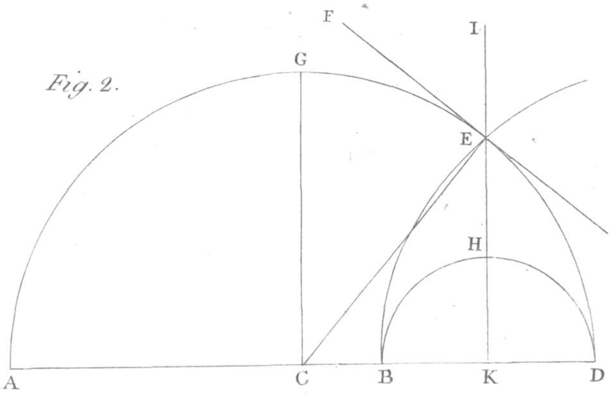


Fig. 3.

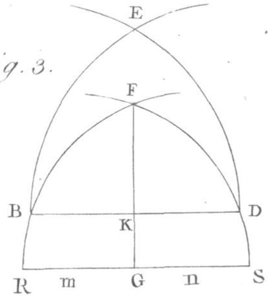


Fig. 4.

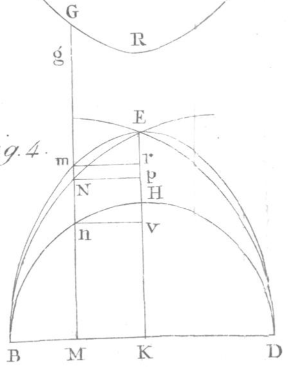


Fig. 5.

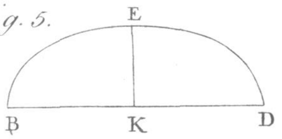
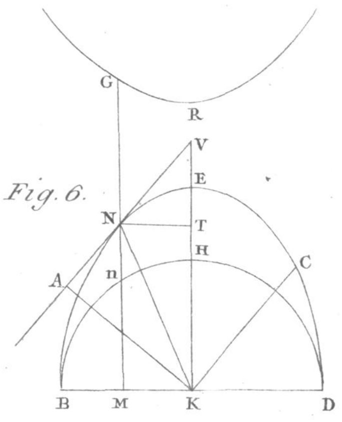


Fig. 6.



J. [illegible] Sculp.